Mathematics Workshop: Crafting Explanations

数学ワークショップ：数式文の作成

MAT314[1] Course Syllabus[2]

> "To everyone who has ever been made to feel bad at mathematics. You didn't fail math: math failed you."
>
> — (Cheng, 2023)

A call for students:

> Take a piece of mathematics. Elementary exercise or an advanced idea, doesn't matter. It's up to you. Spend time with it. A lot. You do have time since you don't need to prepare for an exam. Talk about it, explain it to others. Then, once you own that mathematical idea by your complete understanding, write it down. Craft the best and most beautiful explanation possible. Your final work will be publicly available. Your readers can quickly understand the topic and thus they will have more time work on their piece of mathematical writing. This is how we build knowledge together.

Writing is rewriting. Learning is error-correction.

Introduction

The main purpose of this course is **understanding**. Sadly, school mathematics is obsessed with exam results only. If the student can produce the correct answer, then everyone is happy. No one asks how the solution was found, whether the student knows what is going on, or have no clue. This is the dark deal of mathematics education.

Any standardized test is an invitation for shortcut learning. Students use simple made-up methods for solving a particular type of problem, without grasping the general principle that would be applicable to problems beyond the test. This situation is the exact opposite of the purpose of education: providing insights and discoveries. The pointless effort of a test-focused learner is even encouraged by the advice 'Just memorize and apply the formulas!' Consequently, students can excel in the subject without any insights about its purpose and significance.

The reality is more grim, since we haven't even mentioned the majority of people for whom the correct answer remains elusive. *How can we facilitate understanding then?*

Writing to learn

> "Writing is thinking on paper. Anyone who thinks clearly should be able to write clearly - about any subject at all."
>
> — (Zinsser, 1988)

[1] $314 \approx 100\pi$

[2] This document was created on 2025-06-14. During a semester, some of the content is subject to change (e.g., curated list of books and recommended topics). However, the assessment conditions should remain the same.

We know that there are several levels of understanding something. Teachers often say that they can finally understand something after years of teaching it. Crafting an explanation and writing it down seem to be one of the highest levels of understanding. In this course, we make this into the main method for understanding mathematics. Thus, taking this course needs the transition from being a student to being a researcher. This is an equally challenging, but way more fulfilling role, than a diligent rote learner.

Why Mathematics?

One can easily accept the importance of writing as a learning method, but the question remains: Why spending time on studying Mathematics? The usefulness of Mathematics is undeniable but but putting too much emphasis utility is harmful in education. Most students might say that if mathematics is used for building bridges, designing aircrafts and improving computers, and I'm not doing any of those, then I can skip the subject.

Here, we advocate the usefulness of mathematical thinking on a more general level, applying to everyone. Solving hard problems in the sandbox world of mathematics builds mental stamina, thus it prepares a person to deal with the tough issues of life. Not by applying the quadratic equation, but by being familiar with the feeling of overcoming a difficulty, by the experience of efficient thinking and precise communication.

Typography

The visual quality of written and printed mathematical text is important. It is not some additional aesthetics. It is a core requirement. Mathematics is about communicating difficult ideas, sharing concepts at the edge of our understanding. Therefore, we cannot afford mistakes due to ambiguity of the notation, poor legibility of the symbols. In a novel, we may not even recognize a typo. Mathematical text is the least forgiving. In mathematical writing we mix natural language with symbols. One symbol changes, and the whole meaning is gone.

Consequently, the technical requirements for mathematical publishing are demanding. Ordinary, consumer word processors may not have adequate algorithms for properly typesetting formulas and matrices, aligning equations. Hence the need for professional tools. After all, impressive ideas should be beautiful to look at.

The widely used LaTeX typesetting system takes long time to learn and it is easy to forget the fine details between writing papers. Luckily, we now have an emerging alternative Typst[3], even with an online collaborative tool. This makes it possible to start composing mathematical texts without an extensive preliminary training.

https://typst.app

https://typst.app/docs/

https://github.com/johanvx/typst-undergradmath/releases/latest

Same for LaTeX:

https://tug.ctan.org/info/undergradmath/undergradmath.pdf

Learning Outcomes

1. Improved reasoning and communication skills for explaining complex ideas.
2. Technical knowledge for producing quality professional documents.

[3]This document was created in Typst 0.13.1, using the Libertinus font family.

3. An understanding of the 'cultural disaster' that somehow turned most people away from the art of efficient and precise thinking.
4. Have a more complete view of contemporary mathematics, as opposed to the biased and limited selection of topics in school mathematics.

Requirements

Regarding the mathematics, there is no requirement. School mathematics gives enough background knowledge for starting mathematical writing. The course is listed at the 300-level, since it requires a certain level of maturity (and distancing from the school-exam mindset) to see clearly that mathematical writing is doable by and useful for everyone. It is also assumed that the student has a extensive experience in writing in English, or finished the relevant EAP courses.

A personal laptop computer with the ability of installing applications is needed for the course.

Assessment

There is no exam. The main goal of the course to create explanations for the other students (not for the instructor). The final product will be a mathematical paper. The level of the mathematics and the length of the paper depend on the chosen topic, not predetermined. Attendance is part of the assessment due to the classroom activities.

Item	Grade
Introductory 500-word essay	10%
ClassRoom Activities	30%
Paper	50%
Infinity Room session	10%

introductory/motivation 500-word essay describing the mathematical background of the student, will be read aloud in the class;

classroom activities English - math symbols translations, class summaries, free writing, typesetting exercises, peer reviews (therefore attendance is part of the assessment);

mathematical paper A freely chosen topic, written up as an accessible paper. Upon successful completion the paper will be added to a public collection (for the AIU community). The length depends on the topic.

Infinity Room session A 30-minute interactive event open the community, but with small number of participants. The topic is the same as the paper.

Schedule

1. The map(s) of Mathematics. Reading the introductory essays. Choosing topics.
2. Case studies. Reviewing mathematical texts: accessibility of explanations, aesthetic properties of the text and the interaction of these.
3. Typesetting with Typst, LaTeX. Mathematical typography.
4. Math technical knowledge: set theory, numbers and sequences.
5. Typesetting exercises.
6. Elements of mathematical writing (definitions, theorems, proofs).
7. Math technical knowledge: logic, predicate calculus, functions.
8. Typesetting high-quality scalable drawings.
9. Math technical knowledge: axioms and proofs.

10. Bad culture in mathematics communication and education. ((Thurston, 2006), (Lockhart and Devlin, 2009))
11. Technical knowledge: version control, typesetting presentation slides.
12. Session rehearsals.
13. Peer reviews of first drafts. Progress discussions.
14. Math technical knowledge.
15. Peer reviews and final proofreading.

Textbook

There is no single dedicated textbook for this course. Much of the technical material is freely available in tutorials and manuals of the typesetting software. Lecture notes will be provided to cover general mathematical content.

Recommended Reading

Writing

The development of this course was a long and mostly subconscious process. However, the book *Writing to Learn* (Zinsser, 1988) kickstarted the final rapid phase of writing the syllabus. The book is decades old, still it addresses the current issues we have in liberal education today: the ongoing quest for interdisciplinary learning. Maybe even more starkly after the arrival of large language models.

Mathematics Education

The book *Is Math Real?* (Cheng, 2023) argues that students labeled as not good at math may have a better instinct for mathematical inquiry than their peers. Being puzzled by deep questions and asking questions is better than simply following the instructions of an authority (the math teacher). The direct use of mathematics is limited; the indirect (and possibly future) use is unpredictable. It is better to ask, "Am I developing myself by doing this?" instead of, "Am I going to use this ever?" Emphasis should be on the explanation. "What is 4×6?" is a bad question requiring memorization, but "$4 \times 6 = 24$, why?" asks for an explanation, thus checking understanding. Math is more about building good justifications than getting the correct answer. We can be subjective about the explanations, while the logical truth is indisputable.Asking why doing mathematics quickly leads to why doing anything at all. There are several ways of doing mathematics rooted in different cultures. At the moment, the academic form is dominant and could be deeply connected to a colonial mindset in several ways. However, the most urgent task is to stop inflicting harm on young people in the classrooms.

Typography

Typography is huge and beautiful topic. For a beginner, the short introduction (Luna, 2018) is recommended to get the main ideas about typography.

Mathematical writing

The instructional book (Vivaldi, 2014) for math major undergraduate and graduate students is the closest to being an official textbook for this course. The hefty (Cummings, 2021) advocates *long-form maths*, text including commentary, motivation and explanation. For writing proofs, (Velleman, 2019) provides a structured approach.

Mathematics Books

The Infinity Room hosts a selection of mathematical books. Here we mention a few outstanding books relevant to this course. *Concrete Mathematics - A Foundation for Computer Science* is a legendary textbook; a famous designer created a dedicated typeface for this book. *Mathematics with Bad Drawings*, as the title suggests, creates a unique style with admittedly bad hand drawings. *An invitation to applied*

category theory : seven sketches in compositionality is a prime example of how modern and important mathematics is presented in a consistent and high-quality typographic format. The illustrations in the *Proof and the Art of Mathematics* show the precision and aesthetic value of diagrams created by typesetting programs.

Topic ideas

Here is a small sample of project ideas.

1. Hilbert's Hotel
2. Cryptographic hash-functions
3. Cellular automata
4. QR codes
5. Knot theory
6. Rubik's Cube and group theory
7. What does π do in the normal distribution?
8. Wavefronts and ellipses.
9. Torchlight and conic sections.
10. …

Applied International Liberal Arts AILA

AILA Elements

Mathematics can be viewed as a language used by people who work together on solving difficult problems. As such, it is international. Even nations waging wars against each other would agree on mathematical ideas. This precise and efficient form of communicating, or in other words, the quantitative and computational thinking are essential for facing challenges both on the local and the global level.

The course aims to develop these skills by giving agency to students. It emphasizes

- human-to-human communication and reasoning skills,
- sustained effort in growing knowledge,
- appreciation for aesthetic values.

AILA Activities & Projects

The course is designed for making the productive work in the classroom visible for the AIU community. Therefore, Infinity Room sessions are part of the assessment. Moreover, the final papers will be collected and made available in the Infinity Room and in the Library.

Academic Policies

In addition to the *AIU Academic Dishonesty Policy*, **the undeclared use any generative AIs for writing text is a serious offense leading to failing the course**. Any declared use of generative AIs will lead to zero marks for that assignment. The course values human-to-human communication and considers partaking in and supporting industrial-scale plagiarisms morally questionable.

Logistics

Semester: 2025 Spring

Capacity: 25

Room: D103 or L203

Credits: 3

Bibliography

Cheng, E. (2023) *Is Math Real?: How Simple Questions Lead Us to Mathematics' Deepest Truths*. Basic Books.

Cummings, J. (2021) *Proofs: A Long-form Mathematics Textbook*. LongFormMath.com.

Lockhart, P. and Devlin, K. (2009) *A Mathematician's Lament: How School Cheats Us Out of Our Most Fascinating and Imaginative Art Form*. Bellevue Literary Press.

Luna, P. (2018) *Typography: a Very Short Introduction*. Oxford University Press (Very Short Introductions Series).

Thurston, W.P. (2006) "On Proof and Progress in Mathematics," in R. Hersh (ed.) *18 Unconventional Essays on the Nature of Mathematics*. New York, NY: Springer New York, pp. 37–55. Available at: https://doi.org/10.1007/0-387-29831-2_3.

Velleman, D. (2019) *How to Prove It: A Structured Approach*. Cambridge University Press.

Vivaldi, F. (2014) *Mathematical Writing*. Springer London (Springer Undergraduate Mathematics Series).

Zinsser, W. (1988) *Writing to Learn*. Harper & Row (Perennial library).

Electronic Books

Fortunately, it is part of the mathematical culture to write up some lecture notes and release them for free. Some authors try and succeed to have an agreement with the publisher that an electronic copy of the book is always available for free. Here is a (growing) curated list of free and legal electronic textbooks and mathematical monographs.

Algorithms

1. *Algorithms* by Jeff Erickson, University of Illinois at Urbana-Champaign, 1st Edition 2019. http://jeffe.cs.illinois.edu/teaching/algorithms/

Category Theory

1. *Basic Category Theory* by Tom Leinster, University of Edinburgh,2016. https://arxiv.org/pdf/1612.09375
2. *Category Theory for Computing Science* by Michael Barr and Charles Wells, 1998. https://www.math.mcgill.ca/triples/Barr-Wells-ctcs.pdf
3. *Category Theory in Context* by Emily Riehl, Johns Hopkins University, 2016. https://emilyriehl.github.io/files/context.pdf
4. *Generic figures and their glueings, A constructive approach to functor categories* by Marie La Palme Reyes, Gonzalo E. Reyes and Houman Zolfaghari, 2004. https://reyes-reyes.com/2004/06/01/generic-figures-and-their-glueings-a-constructive-approach-to-functor-categories/
5. *Seven Sketches in Compositionality: An Invitation to Applied Category Theory* by Brendan Fong, David I Spivak, 2018. https://arxiv.org/pdf/1803.05316
6. *Sheaf Theory through Examples* by Daniel Rosiak, 2022. https://direct.mit.edu/books/oa-monograph-pdf/2368025/book_9780262370424.pdf

Combinatorics

1. *generatingfunctionology 2nd edition* by Herbert S. Wilf, University of Pennsylvania, 1994. https://www2.math.upenn.edu/~wilf/DownldGF.html

Complex Analysis

1. *Complex Analysis: An Open Source Textbook* by Russell W. Howell, Westmont College, 2025. https://complexanalysis.org/

Cryptography

1. *Joy of Cryptography* by Mike Rosulek, Oregon State University, https://joyofcryptography.com/

Games

1. *The Raven's Hat – Fallen Pictures, Rising Sequences, and Other Mathematical Games* by Jonas Peters (University of Copenhagen) and Nicolai Meinshausen from (ETH Swiss Federal Institute of Technology). https://mitpress.mit.edu/9780262044516/the-ravens-hat/

Graph Theory

1. *Graph Theory* by Reinhardt Diestel, University of Hamburg, 2025, https://diestel-graph-theory.com/basic.html

Linear Algebra

1. *Linear Algebra Done Right* by Sheldon Axler, San Francisco State University, 2024. https://linear.axler.net/

Logic

1. *Logic Matters* https://www.logicmatters.net/books/
2. *Open Logic Textbooks* (several textbooks on beginner and advanced logic and the theory of computation) https://builds.openlogicproject.org/

Number Theory

1. *Topology of Numbers* by Allen Hatcher, 2022. https://pi.math.cornell.edu/~hatcher/TN/TNbook.pdf

Probability

1. *Introduction to Probability* by Charles Grinstead & J. Laurie Snell, Swarthmore, Dartmouth Colleges, 2006. https://math.dartmouth.edu/~prob/prob/prob.pdf

Proofs

1. *Introduction to Proof via Inquiry-Based Learning* by Dana C. Ernst, Northern Arizona University, 2022. https://danaernst.com/IBL-IntroToProof/
2. *Proof Book 3rd Edition* by Richard Hammack, Virgina Commonwealth University, 2022. https://richardhammack.github.io/BookOfProof/

Topology

1. *Algebraic Topology* by Allen Hatcher, 2002. https://pi.math.cornell.edu/~hatcher/AT/ATpage.html
2. *Topology and Groupoids* by Ronald Brown, 2006. https://groupoids.org.uk/topgpds.html